HN-003-016304 Seat No. ________

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

May / June – 2017

MATH.CMT - 3004 : Discrete Mathematics

Faculty Code : 003
Subject Code : 016304

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all the questions.
(2) Each question carries 14 marks.

1 Answer any **Seven** : **7×2=14**

(a) Verify that $00^*(0\vee 1)^* 1$ is a regularr expression over $\{0.1\}$.

(b) Let A be a $m\times n$ Boolean matrix and B be a $n\times p$ Boolean matrix. Define the *Boolean product* of A and B.

(c) Let R be the relation defined on $\mathbb{Z}\times\mathbb{Z}$ by $(a, b)R(x, y)$ if and only if $a-b=x-y$. Prove that R is an *eguivalence relation* on $\mathbb{Z}\times\mathbb{Z}$.

(d) If $(P, \leq)$ is a partially ordered set, then prove that $(P, \leq^{-1})$ is a partially ordered set.

(e) Define a *modular lattice* and illustrate it with an example.

(f) Define a *Boolean Algebra* Let $n\geq 1$. Show that there exists a Boolean Algebra containing exactly 2^n elements.

(g) Define *homomorphism of semigroups*. Let $(S.*)\to(T.*')$ be a surjective homomorphism of semigroups. If $(S.*)$ is a monoid, then show that $(T.*')$ is also a monoid.

(h) Let $I=\{a,b\}$ and $L\subseteq I^*$, When is L said to be a *type three language* ?

(i) Define a *machine congruence* on a finite state machine.

(j) Define the *direct derivability* relation associated with a phrase structure grammar G.

2 Answer any **Two** : **2×7=14**

(a) Let H be a normal subgroup of a group G. Define the relation R on G by aRb if and only if $ab^{-1} \in H$. Prove that R is a congruence relation on G.

(b) State and prove the *fundamental theorem of homomorphism of semi-groups.*

(c) Let $n \geq 1$. Show that D_n. the lattice of positive divisors of n is distributive.

3 (a) Let A be a finite nonempty set. Define the concept of *regular expression ouer A.* **5**

(b) Let $(L_i, \leq_i)$ be lattices for each $i \in \{1,.....n\}$, Let $L = L_1 \times .... \times L_n$. Prove that $(L, \leq)$ is a lattice, where $\leq$ is the product partial order on L. **5**

(c) Let *p,q* be propositions. Construct a truth table for the statement $(p \wedge q) \vee \tilde{p}$. is the negation of P. **4**

OR

3 (a) Let R be a relation defined on a nonempty set A. Prove that R^{∞} is the transitive closure of R. **5**

(b) Let $f : L_1 \rightarrow L_2$ be a bijection, where $(L_i, \leq_i)$ is a lattice for each $i \in \{1,2\}$. Show that $f(a \vee b) - f(a) \vee f(b)$ for all $a, b \in L_1$ if and only if f and f^{-1} both preserve order. **5**

(c) Construct the truth table for the Boolean function $f : B_3 \rightarrow B$ determined by the Boolean polynomial $p(x_1.x_2.x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$. **4**

4 Answer any **Two** : **2×7=14**

(a) Let $(L, \leq)$ be a finite Boolean Algebra. Let $a \in L, a \neq 0$. Let $W = \{b \in L : b$ is an atom of $(L. \leq)$ with $b \leq a\}$. Prove that $a = v_{b \in W} b$.

(b) State and prove the Pumping lemma.

(c) Let M be a Moore machine with $S=\{s_0,s_1.s_2\},I=\{a,b\}.\ f_a:S\rightarrow S$ equals the identity mapping on $S.f_b(s_0)=s_1,\ f_b(s_1)=s_2.f_b(s_2)=s_2$ and $T=\{s_2\}$. Find $L(M)$ and also determine a regular expression α over I such that L (M) is the regular set corresponding to α.

5 Answer any **Two** : **2×7=14**

(a) Let M be a Moore machine with S as its state set. Prove that the w-compatibility relation R defined on S is a machine congruence on M and $L(M)=L(M/R)$.

(b) Let $n\geq 1$ Prove that any finction $f:B_n\rightarrow B$ is produced by a Boolean expression.

(c) Let $(L.\leq)$ be a lattice. Show that $(L.\leq)$ is distributive if and only if for all $a,b,c\in L,(a\wedge b)\vee(b\wedge c)\vee(c\wedge a)=(a\vee b)\wedge(b\vee c)\wedge(c\vee a)$.

(d) Let M be a Moore machine with I as its input set. Show that there exists a type three phrase structure grammar G with I as its set of terminal symbols such that $L(M)=L(G)$.